Randomized Response Techniques for Multiple Sensitive Attributes

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Some randomized response techniques for investigating $t \geqq 2$ sensitive attributes are reviewed. A new technique is proposed that has the advantage of requiring only r trials per respondent ($r \leqq t$) if estimation of up to r-variate joint proportions is desired. The case of $r = 2$ is analyzed in detail. A procedure for deriving the restricted maximum likelihood estimators (MLE's) of the proportions and a test of independence between any set of pairs of attributes are given. The notion of measure of respondent jeopardy is extended to our setup. Keeping this measure fixed, we make numerical comparisons for the $t = 2$ case between competing techniques in terms of the trace of the asymptotic variance-covariance matrix of the estimator vector. Finally, a practical application of the new technique is described.

KEY WORDS: Randomized response; Restricted maximum likelihood estimators; Multiple sensitive attributes; Sample survey techniques; Respondent jeopardy function.

1. INTRODUCTION

Surveys for eliciting information on sensitive or stigmatizing attributes are plagued by the problem of untruthful responses or noncooperation by respondents, both of which lead to biased estimates. To avoid this "evasive answer bias" and to preserve the privacy of the respondent, Warner (1965) introduced an innovative technique commonly referred to as randomized response (RR) technique. Since Warner's article, many authors have made contributions to this general area; a review of these contributions may be found in Horvitz, Greenberg, and Abernathy (1975).

Most of the work on RR techniques is restricted to the study of a *single* sensitive attribute. Very often, however, social researchers are interested in studying several sensitive attributes together. Thus the researchers are not only interested in estimating and testing hypotheses concerning the proportions of the population possessing the individual sensitive attributes under study, but also the degree of association between the different attributes. Suitable statistical techniques for collecting and analyzing data for surveys dealing with such multiple sensitive attributes do not appear to be available.

The purpose of this paper is two-fold. First, we briefly review some recent literature that has a bearing on the multiple sensitive attributes problem. Second, we propose and develop for the given problem a new RR technique that has some desirable properties. This technique is an extension of a technique earlier proposed by Barksdale (1971), but the estimation procedure proposed here is new. We also give a test of pairwise independence for any set of pairs of attributes. We extend the notion of respondent jeopardy proposed by Leysieffer and Warner (1976) to the multiple sensitive attributes set up. Keeping this measure of respondent jeopardy fixed, we carry out a numerical comparison of efficiencies of some competing procedures. Finally, we give the results of an actual application of the technique to demonstrate its feasibility in practice.

In the numerical comparisons it turns out that the proposed technique does not fare as well as an "optimal" version of a technique involving a repeated (for each attribute) application of the Simmons unrelated question technique. Nevertheless, it was felt desirable to publish the results because the technique does have some practical advantages and performs at least reasonably well. In any case, the comparisons between competing techniques should prove useful to the practitioner. Furthermore, many of the results are new and interesting and it is hoped that they will attract other researchers to work on the problem.

2. SOME PREVIOUS WORK—AN OVERVIEW

In his dissertation, Barksdale (1971) proposed and analyzed some RR techniques for investigating *two* sensitive dichotomous attributes. In particular, he considered a repeated (for each attribute) application of Warner's original technique (see also Clickner and Iglewicz 1976), a repeated application of Simmons's unrelated question technique (Greenberg et al. 1969), and a third technique that we describe in detail in the next section. In the repeated application of Warner's technique (*W technique*) two trials are performed per respondent. On the ith trial ($i = 1, 2$) the interviewer presents the respondent with

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a pair of statements: "I possess the attribute A_i" and "I do not possess the attribute A_i," where A_i is a sensitive attribute. The respondent picks one of the two statements at random according to known probabilities P_i and $1 - P_i$ ($P_i \neq \frac{1}{2}$) and, without revealing his choice to the interviewer, responds to it. Then from the observed frequencies of "Yes-Yes," "Yes-No," "No-Yes," and "No-No" responses, and the knowledge of the P_i's, the desired proportions can be estimated. The repeated application of Simmons's technique (*S technique*) is quite similar, except that on the ith trial ($i = 1, 2$) the respondent is presented with a pair of statements, "I possess the attribute A_i" and "I possess the attribute Y_i," where Y_i is some unrelated and innocuous attribute. From the knowledge of P_i = the probability of picking the first statement, β_i = the proportion of population possessing the attribute Y_i, and the observed frequencies of responses, the desired proportions can be estimated.

Some other contributions to the problem of multiple sensitive attributes are as follows. Drane (1975, 1976) studied the problem of testing independence between two sensitive dichotomous attributes, using repeated applications of various RR techniques for single attributes. Warner (1971) proposed a general linear RR model for many attributes but did not explicitly consider the problem of joint distributions of the attributes. Another technique for estimating marginal distributions of several sensitive attributes that makes use of weighing designs was proposed by Raghavarao and Federer (1979).

Related work on the RR techniques for multiple sensitive attributes has been done in Europe by Eriksson (1973) and Bourke (1975). Eriksson presented a theory for the general case of a two-way contingency table. Bourke considered various designs for estimating the corresponding cell probabilities in a two-way table formed by t sensitive attributes, each having c categories of which at most $(c - 1)$ are sensitive. Bourke's work does not, however, address the problem of estimating joint proportions of different attributes. The details of some of these techniques are found in Horvitz, Greenberg, and Abernathy (1976).

3. MULTIPLE RR TRIALS TECHNIQUE

3.1 Barksdale's Third Technique

The technique we are about to propose is an extension of the third technique proposed by Barksdale (1971), which is as follows. The two statements concerning the two sensitive attributes are phrased so that a "Yes" response to one of the two statements would be nonstigmatizing (e.g., the two statements might be "I have never smoked marijuana" and "I am an alcoholic"). The interviewer presents both statements to the respondent on two occasions. On each occasion the respondent picks one of the two statements at random, unknown to the interviewer, but according to some known probability (different for each occasion), and responds to it. This procedure maintains the privacy of the respondent and yet allows the researcher to compute the estimates of the marginal and bivariate proportions of the attributes from the observed frequencies of "Yes-Yes," "Yes-No," "No-Yes," and "No-No" responses.

In a survey dealing with $t \geq 2$ sensitive attributes, the W and S techniques involve t trials per respondent. If t is large, then these techniques become tedious, costly, and lead to degradation in cooperation on the part of the respondents. Also, the estimating equations involve *all* the joint proportions, which often the researcher is not interested in. On the other hand, the technique described in the previous paragraph can be easily extended to the case of $t > 2$, with the number of trials per respondent restricted to $r < t$ if the researcher's interest only lies in up to r-variate joint proportions. Quite often, $r = 2$ will suffice for the purposes of the research.

Intuitively, it is clear that for $t > 2$, since the W and S techniques involve t trials while the technique to be proposed involves only $r < t$ trials, the latter technique must be less informative. This is indeed so. Part of the extra information obtained by the former techniques is in the form of estimates of higher order joint proportions that are not obtainable with the latter technique, while the rest of the extra information manifests itself in terms of lower variances of the estimates. The former techniques, however, would suffer from degradation in cooperation for t as low as three or four while the latter technique, for fixed r (which is based on investigator's interests and goals) would suffer from somewhat inflated variances. The exact trade-off is not clear, nor is it clear how much larger sample sizes would be required with the latter technique to compensate for the inflated variances. These issues need further research.

Now we describe the latter technique, which we refer to as the *multiple RR trials technique* or the *M technique*.

3.2 Notation and Description of the Technique

Consider $t \geq 2$ dichotomous attributes $A_1, A_2, \ldots, A_t$; we shall assume that all the attributes are sensitive, but obviously that need not be so. Let $\theta_{i_1 \ldots i_u}$ denote the unknown proportion of individuals in the target population that possess the attributes $A_{i_1}, \ldots, A_{i_u}$ ($1 \leq i_1 < \cdots < i_u \leq t$, $1 \leq u \leq t$). The researcher's interest lies in making statistical inferences (estimation and hypothesis testing) concerning the θ's.

For employing the multiple RR trials technique, the statements must be phrased so that a "Yes" response to some statements would be nonstigmatizing, while a "No" response to the others would be so. Without loss of generality, we shall assume that the first $s < t$ statements are phrased "I possess the attribute A_i" ($1 \leq i \leq s$), a "No" response to each one of which would be nonstigmatizing; the remaining $t - s$ statements are phrased "I do not possess the attribute A_i" ($s + 1 \leq i \leq t$), a "Yes" response to each one of which would be so. An appropriate choice of s would be $\cong t/2$. Let $\pi_{i_1 \ldots i_u}$ be defined in the same manner as $\theta_{i_1 \ldots i_u}$ but with respect to the modified attributes B_i, which are either the original $A_i(1 \leqq i \leqq s)$ or the complements of the $A_i(s + 1 \leqq i \leqq t)$. It is clear that the θ's can be obtained from the π's and vice versa, and therefore we shall consider the equivalent problem of estimation of the π's.

As remarked in the previous section, we shall assume that the researcher is interested only in the marginal and bivariate proportions; that is, $\pi_i(1 \leqq i \leqq t)$ and $\pi_{ij}(1 \leqq i < j \leqq t)$, respectively. Thus there are $t(t + 1)/2$ unknown parameters to be estimated and only two trials may be performed per respondent. We now describe the technique.

A total sample of n individuals is divided into $b \geqq 1$ subsamples; the value of b will be specified in the following section. Let $n_1, n_2, \ldots, n_b$ be the subsample sizes with $\sum_{h=1}^{b} n_h = n$.

Each individual is presented all the t statements and asked to respond to one statement picked at random according to some randomizing device, but not reveal his choice of the statement to the interviewer. This procedure is repeated with another randomizing device, and both the responses are recorded. Let $P_{hi}^{(l)}$ denote the (known) probability that an individual drawn from the hth subsample picks, on the lth trial, the ith statement $(1 \leqq i \leqq t)$; obviously we have $\sum_{i=1}^{t} P_{hi}^{(l)} = 1$ for $1 \leqq h \leqq b$ and $l = 1, 2$.

3.3 Estimation of the π's

Suppose that the responses are coded so that a score of 2^{l-1} is assigned to a "Yes" response on the lth trial and a score of 0 is assigned to a "No" response. Then the total score, say ν, completely identifies the individual's response. For example, $\nu = 3$ corresponds to a "Yes-Yes" response, $\nu = 2$ corresponds to a "No-Yes" response, and so on. Let $\lambda_{h\nu}$ denote the probability of obtaining a score of ν for an individual drawn from the hth subsample, $\boldsymbol{\lambda} = (\lambda_{11}, \lambda_{12}, \lambda_{13}, \ldots, \lambda_{b1}, \lambda_{b2}, \lambda_{b3})'$, and $\boldsymbol{\pi} = (\pi_1, \ldots, \pi_t, \pi_{12}, \pi_{13}, \pi_{t-1,t})'$. Then we have

$$\boldsymbol{\lambda} = \mathbf{R}\boldsymbol{\pi}, \tag{3.1}$$

where the elements of the matrix $\mathbf{R} = \{R_{ij}\}$ are given by the following equations. For $1 \leqq h \leqq b$ and $1 \leqq i \leqq t$ we have

$$\begin{aligned} R_{3h-2,i} &= P_{hi}^{(1)}(1 - P_{hi}^{(2)}), \\ R_{3h-1,i} &= P_{hi}^{(2)}(1 - P_{hi}^{(1)}), \\ R_{3h,i} &= P_{hi}^{(1)}P_{hi}^{(2)}, \end{aligned} \tag{3.2}$$

and for $1 \leqq i < j \leqq t$ if $k = it - i(i + 1)/2 + j$ we have

$$\begin{aligned} R_{3h-2,k} &= -(P_{hi}^{(1)}P_{hj}^{(2)} + P_{hj}^{(1)}P_{hi}^{(2)}) \\ &= R_{3h-1,k} = -R_{3h,k}. \end{aligned} \tag{3.3}$$

To find b, the total number of subsamples necessary to estimate the t marginal proportions $\{\pi_i\}$ and $\binom{t}{2}$ bivariate proportions $\{\pi_{ij}\}$, consider an extreme case (and a most favorable one from the statistician's viewpoint) in which the P values can be chosen either equal to zero or one (which corresponds to the "direct response" case). By choosing $P_{hi}^{(1)} = 1$ and $P_{hj}^{(2)} = 1$ for different pairs (i, j) for different subsamples $h(1 \leq h \leq b)$, it is easy to see that all the parameters can be estimated by using $\binom{t}{2}$ subsamples, and no smaller number of subsamples will do. An extension of this argument shows that even for general P values, at least $\binom{t}{2}$ subsamples are required to estimate all the parameters. In other words, by suitably choosing the P's, the matrix $\mathbf{R}$ defined in (3.2) and (3.3) can be made to have a full column rank only if $b \geqq \binom{t}{2}$. Let us then assume that $b \geqq \binom{t}{2}$ and that $\mathbf{R}$ is a full column rank matrix.

We propose to obtain the maximum likelihood estimator (MLE) of $\boldsymbol{\pi}$ from the observed data $\{n_{h\nu}\}$ where $n_{h\nu}$ = the number of individuals from the hth subsample having a score of $\nu(0 \leqq \nu \leqq 3)$, $\sum_{\nu=0}^{3} n_{h\nu} = n_h(1 \leqq h \leqq b)$. The usual method of first obtaining the *unrestricted* MLE (UMLE) of $\boldsymbol{\lambda}$ (i.e., the UMLE of $\lambda_{h\nu} = n_{h\nu}/n_h$ for $0 \leqq \nu \leqq 3$, $1 \leqq h \leqq b$) and then obtaining the UMLE of $\boldsymbol{\pi}$ by "solving" (3.1) is not applicable for two reasons in the present context.

1. Matrix $\mathbf{R}$ can be chosen to be a *square* full rank matrix only for $t = 2$. For $t > 2$, in general, there is no unique solution in $\boldsymbol{\pi}$ to (3.1).
2. Even in the case in which the UMLE of $\boldsymbol{\pi}$ can be obtained by the above method, the resulting estimator may not satisfy the natural restrictions on the π's, namely, that

$$\begin{aligned} 0 \leqq \pi_i \leqq 1 \quad &\forall i \text{ and} \\ \max(0, \pi_i + \pi_j - 1) \leqq \pi_{ij} \leqq \min(\pi_i, \pi_j) \quad &\forall (i, j). \end{aligned} \tag{3.4}$$

From a theoretical viewpoint, the UMLE of $\boldsymbol{\pi}$ may even be inadmissible, as shown in the case of Warner's technique for a single attribute by Fligner, Policello, and Singh (1977) and Devore (1977); it appears that Warner (1965) was also aware of this problem, as is evident from the footnote on page 65 of his paper.

Therefore, we must find the restricted MLE (RMLE) of $\boldsymbol{\pi}$, say $\hat{\boldsymbol{\pi}}$. We propose to obtain $\hat{\boldsymbol{\pi}}$ directly by maximizing the likelihood function

$$L \propto \prod_{h=1}^{b} \prod_{\nu=0}^{3} (\lambda_{h\nu})^{n_{h\nu}} \tag{3.5}$$

subject to (3.4). In (3.5) the $\lambda_{h\nu}$ are given in terms of $\boldsymbol{\pi}$ by (3.1). Denote the restricted maximum of L by L^*. The constraint set (3.4) is linear in the π's and the objective function $\log_e L$ can be easily checked to be concave in the π's. The resulting nonlinear programming problem is thus well structured and can be solved quite economically on a computer using one of the commonly available algorithms.

3.4 Properties of $\hat{\boldsymbol{\pi}}$

The RMLE $\hat{\boldsymbol{\pi}}$ is biased in small samples but is asymptotically (as $n_h \to \infty$, $\forall h$) unbiased. The asymptotic variance-covariance matrix of $\hat{\boldsymbol{\pi}}$ (which is also the exact variance-covariance matrix of the UMLE of $\boldsymbol{\pi}$) is given by the inverse of the information matrix $\mathcal{I}$; we give below an expression for the elements of the upper left $t \times t$ principal submatrix of $\mathcal{I}$. For $1 \le i, j \le t$ we have

$$\mathcal{I}_{ij} = -E\{\partial^2 \log L/\partial\pi_i\partial\pi_j\} = \sum_{h=1}^{b} n_h \sum_{\nu=0}^{3} (1/\lambda_{h\nu})(\partial\lambda_{h\nu}/\partial\pi_i)(\partial\lambda_{h\nu}/\partial\pi_j).$$

The remaining elements of $\mathcal{I}$, which would involve $\partial\lambda_{h\nu}/\partial\pi_{ij}$ terms, can be obtained in an analogous manner. The various derivatives can be evaluated easily by using (3.1).

Expressions for the variances and covariances of the RMLE's of the θ's, say $\hat{\theta}$'s, can be obtained from those of the π's. Large-sample hypothesis testing concerning the θ's can be carried out by using the expressions for their asymptotic variances, with $\boldsymbol{\lambda}$ replaced by its consistent estimate $\hat{\boldsymbol{\lambda}} = \mathbf{R}\hat{\boldsymbol{\pi}}$. Expressions for the (asymptotic) variances for $t = 2$ are not given here but are obtainable from the author.

3.5 Test of Independence

First we note that testing pairwise independence between the original attributes, say A_i and A_j, is equivalent to testing pairwise independence between the corresponding modified attributes. Therefore, we shall consider the problem of testing independence between pairs of modified attributes.

Suppose that it is desired to test the hypothesis $H_{\mathcal{F}}$: $\pi_{ij} = \pi_i\pi_j$ for all pairs (i, j) in a certain set $\mathcal{F}$. We can use the *generalized likelihood ratio* method to test this hypothesis as follows. Compute the maximum of the likelihood function L in (3.5) subject to the following constraints on the π's

$$\begin{aligned} 0 \le \pi_i \le 1 \quad &\forall i, \\ \max(0, \pi_i + \pi_j - 1) \le \pi_{ij} \le \min(\pi_i, \pi_j) \quad &\forall (i, j) \notin \mathcal{F} \\ \pi_{ij} = \pi_i\pi_j \quad &\forall (i, j) \in \mathcal{F}. \end{aligned} \tag{3.6}$$

Denote the corresponding maximum value of L by $L_{\mathcal{F}}^*$. Then under $H_{\mathcal{F}}$ asymptotically $-2 \log_e(L_{\mathcal{F}}^*/L^*)$ has a chi-squared distribution with f degrees of freedom (df), where f is the number of pairs in the set $\mathcal{F}$.

3.6 Choice of $\{P_{hi}^{(l)}\}$

The determination of the "optimal" (for an appropriate criterion and subject to suitable constraints on the respondent jeopardy; see Sec. 5.1) choice of the design probabilities $\{P_{hi}^{(l)}\}$ appears to be a difficult problem because of the complexities of the expressions for the asymptotic variance-covariances of $\{\hat{\pi}\}$ and the number of design parameters that can be manipulated. It should be pointed out that even if expressions for "optimal" $\{P_{hi}^{(l)}\}$ can be obtained, they would depend on the unknown vector $\boldsymbol{\pi}$. Thus, for implementation purposes one must use some prior estimate of $\boldsymbol{\pi}$.

Because of the above difficulties, we provide only some heuristic guidelines for the choice of $\{P_{hi}^{(l)}\}$. It can be readily verified that if each $P_{hi}^{(l)} = 1/t$, then matrix $\mathbf{R}$ in (3.1) becomes a deficient column rank matrix and hence $\boldsymbol{\pi}$ is not estimable. Therefore, for fixed h and l, the $P_{hi}^{(l)}$ should be chosen as far away (in either direction) from $1/t$ as possible, subject to some respondent jeopardy constraint and the constraint that $\sum_{i=1}^{t} P_{hi}^{(l)} = 1$ for $1 \leqq h \leqq b$ and $l = 1, 2$. In fact, for large t, the length of the questionnaires can be cut down for different subsamples by choosing $P_{hi}^{(l)} = 0$ for different sets of statements. If the researcher is equally interested in all the attributes, the $P_{hi}^{(l)}$ should be chosen symmetrically as far as possible. For $t = 2$, such a symmetric choice is provided by $P_{11}^{(1)} + P_{11}^{(2)} = 1$; subject to this restriction, $P_{11}^{(1)}$ and $P_{11}^{(2)}$ may be chosen as far away from $\frac{1}{2}$ as the jeopardy constraint permits. The choice will depend on the average educational and social sophistication of the population. A pilot survey should be carried out to test different randomizing devices (different $\{P_{hi}^{(l)}\}$), as well as the questionnaire itself.

4. A MEASURE OF RESPONDENT JEOPARDY

We shall consider two techniques in competition with the M technique developed here: the W technique and the S technique. For a fair comparison between these techniques it is necessary to keep some measure of the jeopardy of respondent's privacy fixed. In the following section we develop such a measure.

4.1 Definition of the Jeopardy Function

Leysieffer and Warner (1976) and Lanke (1976) have developed two different approaches for constructing such measures. Here we shall extend only the Leysieffer-Warner approach to the case of $t \geq 2$ sensitive attributes. The Lanke approach can be extended in the same manner, but because of lack of space we do not do so here; the Lanke approach leads to the same choice of design constants for different techniques as the Leysieffer-Warner approach.

Consider the 2^t mutually exclusive and collectively exhaustive groups into which the population is divided depending on the possession or nonpossession of different attributes, and denote these groups by $A_1A_2 \ldots A_t$, $A_1^cA_2 \ldots A_t$, $\ldots$, $A_1^cA_2^c \ldots A_t^c$ where the notation is obvious. Consider, say, the group $A_1A_2 \ldots A_t$. By using Bayes' theorem in the same manner as Leysieffer and Warner (1976), we can show that a measure of information resulting from response ν in favor of $A_1A_2 \ldots A_t$ against $(A_1A_2 \ldots A_t)^c$ is given by

$$g(\nu; A_1A_2 \ldots A_t) = P(\nu \mid A_1A_2 \ldots A_t)/P(\nu \mid (A_1A_2 \ldots A_t)^c). \tag{4.1}$$

Thus the response ν can be regarded as jeopardizing with respect to the group $A_1A_2 \dots A_t$ (and not jeopardizing with respect to $(A_1A_2 \dots A_t)^c$) if $g(\nu; A_1A_2 \dots A_t) > 1$; and not jeopardizing with respect to either $A_1A_2 \dots A_t$ or $(A_1A_2 \dots A_t)^c$ if $g(\nu; A_1A_2 \dots A_t) = 1$. Now to get a measure of the worst jeopardy of the privacy of an individual in group $A_1A_2 \dots A_t$ we define the *jeopardy function* for that group as

$$g(A_1A_2 \dots A_t) = \max_{\nu} g(\nu; A_1A_2 \dots A_t). \quad (4.2)$$

The jeopardy functions for other groups are defined in an identical manner.

The design constants of each RR technique should be chosen so that the jeopardy function values for different groups do not exceed some prespecified upper bounds. We note here that these jeopardy function values will depend in general on the unknown θ's (in contrast to the case of $t = 1$). Therefore, some a priori guesses at values of θ's will be necessary to compute them.

4.2 Jeopardy Functions for the Competing Techniques

Using the definitions (4.1) and (4.2), we shall derive the expressions for the jeopardy functions associated with the W, S, and M techniques for $t = 2$. Here we shall consider only the following special case of practical interest. (The general case with $t \geqq 2$ is quite straightforward but algebraically messy and is hence omitted.) For the W technique we take $P_1 = P_2 = P_W$ (say) where $P_W > \frac{1}{2}$ without loss of generality. For the S technique we take $P_1 = P_2 = P_S$ (say) and $\beta_1 = \beta_2 = \beta$ (say). For the M technique we take $P_{11}^{(1)} = 1 - P_{11}^{(2)} = P_M$ (say) where $P_M > \frac{1}{2}$ without loss of generality.

Define additional notation as follows: $Q_W = 1 - P_W$, $Q_S = 1 - P_S$, $Q_M = 1 - P_M$, $\gamma = 1 - \beta$, and $\theta_{12}^* = 1 - \theta_1 - \theta_2 + \theta_{12}$. The expressions for the jeopardy functions are given in Table 1; the derivations of these expressions are obtainable from the author.

4.3 Equating the Jeopardy Functions for the Competing Techniques

Our approach here will be to first equate the jeopardy functions for the four different groups for the competing techniques and obtain their equivalent design constants, that is, their P values and the β value for the S technique. Clearly, the values of design constants yielded by the four sets of equations will not in general be consistent. We shall follow the convention of guarding the individuals in the most sensitive group, that is, controlling $g(A_1A_2)$ for each technique. The next step in our approach will be to compute for each technique a measure of its performance based on these values of design constants. We have taken the measure of performance to be the trace of the asymptotic variance-covariance matrix of the estimator vector. For $t = 2$, the expressions for the variances of $\hat{\pi}_1$, $\hat{\pi}_2$, and $\hat{\pi}_{12}$ using the three techniques are too lengthy to be given here but are obtainable from the author. These expressions are used in the numerical comparisons carried out in Section 5.

Equating $g_W(A_1A_2)$ with $g_M(A_1A_2)$, we see that

$$P_M = \{\theta_{12} g_W(A_1A_2)\}^{1/2} / [\{\theta_{12}^* g_W(A_1A_2)\}^{1/2} + (1 - \theta_{12})^{1/2}] \quad (4.3)$$

if $g_W(A_1A_2) \geq (1 - \theta_{12})/\theta_{12}^*$. Similar expressions for P_M can be obtained by equating $g(A_1{}^cA_2)$, $g(A_1A_2{}^c)$, and $g(A_1{}^cA_2{}^c)$ for the W and M techniques, but these are not given here. It should be noted that the M technique cannot match the W technique (and also the S technique) at low levels of $g_W(A_1A_2)$; that is, the two techniques would be matched in terms of their jeopardy values for the A_1A_2 group only if $g_W(A_1A_2)$ is not smaller than $(1 - \theta_{12})/\theta_{12}^*$.

Next, equating $g_W(A_1A_2)$ and $g_S(A_1A_2)$, we obtain

$$P_S = \beta(2P_W - 1)/[(1 - P_W) + \beta(2P_W - 1)] \quad (4.4)$$

Thus we have a class of S techniques available with design constants (P_S, β) satisfying (4.4). From this class we can make an optimal choice by selecting that combination (P_S, β) which minimizes the trace of the (asymptotic)

Table 1. Expressions for Jeopardy Functions

	W Technique	S Technique	M Technique
$g(A_1A_2)$	$P_W^2(1 - \theta_{12})/\{P_WQ_W(1 - \theta_{12} - \theta_{12}^*) + Q_W^2\theta_{12}^*\}$	$(P_S + Q_S\beta)^2(1 - \theta_{12})/\{Q_S\beta(P_S + Q_S\beta)$ $\times (1 - \theta_{12} - \theta_{12}^*) + Q_S^2\beta^2\theta_{12}^*\}$	$P_M^2(1 - \theta_{12}/Q_M^2\theta_{12}^*$
$g(A_1{}^cA_2)$	$P_W^2(1 - \theta_2 + \theta_{12})/\{P_WQ_W(\theta_{12} + \theta_{12}^*)$ $+ Q_W^2(\theta_1 - \theta_{12})\}$	$(P_S + Q_S\beta)(P_S + Q_S\gamma)(1 - \theta_2 + \theta_{12})$ $\div \{Q_S\gamma(P_S + Q_S\beta)\theta_{12} + Q_S^2\beta\gamma(\theta_1 - \theta_{12})$ $+ Q_S\beta(P_S + Q_S\gamma)\theta_{12}^*\}$	$(1 - \theta_2 + \theta_{12})/P_MQ_M(\theta_{12} + \theta_{12}^*)$
$g(A_1A_2{}^c)$	$P_W^2(1 - \theta_1 + \theta_{12})/\{P_WQ_W(\theta_{12} + \theta_{12}^*)$ $+ Q_W^2(\theta_2 - \theta_{12})\}$	$(P_S + Q_S\beta)(P_S + Q_S\gamma)(1 - \theta_1 + \theta_{12})$ $\div \{Q_S\gamma(P_S + Q_S\beta)\theta_{12} + Q_S^2\beta\gamma(\theta_2 - \theta_{12})$ $+ Q_S\beta(P_S + Q_S\gamma)\theta_{12}^*\}$	$(1 - \theta_1 + \theta_{12})/P_MQ_M(\theta_{12} + \theta_{12}^*)$
$g(A_1{}^cA_2{}^c)$	$P_W^2(1 - \theta_{12}^*)/\{P_WQ_W(1 - \theta_{12} - \theta_{12}^*) + Q_W^2\theta_{12}\}$	$(P_S + Q_S\gamma)^2(1 - \theta_{12}^*)/\{Q_S\gamma(P_S + Q_S\gamma)$ $\times (1 - \theta_{12} - \theta_{12}^*) + Q_S^2\gamma^2\theta_{12}\}$	$P_M^2(1 - \theta_{12}^*)/Q_M^2\theta_{12}$

variance-covariance matrix of the associated estimator vector. The actual analytical minimization problem is messy because of the complexity of the criterion function. However, we can intuit the optimal choice of (P_S, β) by noting that the criterion function should be a decreasing function of P_S for fixed β and that from (4.4) the maximum value of P_S is obtained when $\beta = 1$. Thus the optimal S technique is the repeated application of the so-called forced yes technique (Drane 1975) with $P_S = (2P_W - 1)/P_W$ and $\beta = 1$. If it is desired to have $g_W(\cdot)$ and $g_S(\cdot)$ equal for all the four groups, then we obtain $P_S = 2P_W - 1$ and $\beta = \frac{1}{2}$. For this choice of parameters the criterion functions for the W and S techniques are identical, and therefore the two techniques are equivalent; this extends the corresponding result for $t = 1$ by Leysieffer and Warner (1976).

5. COMPARISON OF COMPETING TECHNIQUES

5.1 Numerical Results

Define the *trace inefficiency* of an RR technique as the ratio of the trace of the (asymptotic) variance-covariance matrix of its estimates for θ_1, θ_2, and θ_{12} to the corresponding quantity for the direct response technique when both the techniques use the same sample size n. This latter quantity is given by $\{\theta_1(1 - \theta_1) + \theta_2(1 - \theta_2) + \theta_{12}(1 - \theta_{12})\}/n$.

For numerical comparisons, 10 (θ_1, θ_2) combinations representing a wide range of these parameters likely to be encountered in practice were selected; we take $\theta_2 \leqq \theta_1$ without loss of generality. For each (θ_1, θ_2) combination three θ_{12} values were selected: $\theta_{12} = 0$, $\theta_2/2$, and θ_2, thus covering the range of admissible values of θ_{12}. For each $(\theta_1, \theta_2, \theta_{12})$ the value of correlation coefficient ρ_{12} was calculated by using the formula

$$\rho_{12} = (\theta_{12} - \theta_1\theta_2)/\sqrt{\theta_1\theta_2(1 - \theta_1)(1 - \theta_2)}.$$

For each $(\theta_1, \theta_2, \theta_{12})$ combination the results corresponding to four P_W values ($P_W = .70, .75, .80, .85$, which represent the range of P_W values commonly used) were calculated, although here only the results for $P_W = .70$ and $.80$ are given; the results for other P_W values are obtainable from the author. For each P_W the corresponding value of P_M was computed by using (4.3). For the S technique the results for two (P_S, β) combinations are given: an optimal combination with $\beta = 1$ and another one with $\beta = .7$; in either case, the P_S value was computed from (4.4). Of course, the results for the W technique correspond to $\beta = .5$. Thus we get a detailed picture of the performance of the S technique for different choices of its design constants. The values of the trace inefficiencies for all three techniques with design constants determined in the above manner were computed and are given in Table 2.

5.2 Discussion of the Results

First, note that the "optimal" S technique with $\beta = 1.0$ dominates the other techniques in all cases studied. When θ_1 and θ_2 are small ($\leq .05$), the M technique dominates the S technique with $\beta = .7$ uniformly in ρ and P_W in all cases studied. When θ_1 and θ_2 are moderate (between .05 and .10), the M technique dominates whenever ρ is not negative or P_W is not too large, or both. Finally, when θ_1 and θ_2 are somewhat large ($> .10$), the M technique dominates the S technique only when ρ is sufficiently large and positive and P_W is not too large or both. The range of values of θ_1, θ_2, ρ, and P_W for which the M technique dominates the W technique is even greater. In many practical situations dealing with two sensitive attributes, θ_1 and θ_2 are in fact likely to be small and the correlation between the attributes is likely to be positive and large. Furthermore, P_W values that are not too large (usually in the range of .7 to .75) are more commonly used. Thus for the parameter values that are likely to be encountered in practice, the M technique does reasonably well, although not optimally well.

6. APPLICATION OF THE M TECHNIQUE

6.1 Description of the Application

To determine the feasibility of the M technique in face-to-face interviews, a study involving an actual application of the technique was carried out. It was not the objective of this small study to compare the practical feasibilities and performances of all the RR techniques discussed in the previous sections; that comparison would have required a larger study and greater resources than were available to us. However, it was decided to include a control group of subjects who would take the direct response interview and who would provide a datum against which the performance of the M technique can be compared with respect to extent of cooperation and truthfulness of responses. Subjects were randomly allocated to the two groups as explained below.

Students in the Spring 1980 Industrial Psychology (IE-C22) class at Northwestern University provided the 152 subjects for the study. Three other students from the same class were recruited and trained to carry out the interviews. Based on discussions with the student counselor and the staff of the University Clinic, the following two issues were identified as relevant, potentially sensitive, and possibly correlated: (a) using hard drugs and (b) seeking psychiatric help. Accordingly, the following two statements were prepared for use in the direct response and the M technique interviews; of course, for the M technique the second statement was presented in the negative form by modifying it with the inclusion of the parenthetical "not."

Statement 1: I presently take or in the past six months have taken at least one of the following drugs on a regular basis, that is, on the average, at least once a week for a month or longer; acid, angel dust, cocaine, heroin, quaaludes, speed, other drugs in the same category. Identify whether you belong to this group by saying "Yes" or "No."

Table 2. Trace Inefficiencies

θ_1	θ_2	θ_{12}	ρ_{12}	P_W	P_M	W Technique	S Technique β = .7	S Technique β = 1.0	M Technique
.05	.025	.0000	−.0367	.70	.6815	62.859	42.387	29.109	33.126
				.80	.7766	16.579	13.743	11.705	12.469
.05	.025	.0125	.3306	.70	.6875	53.792	36.221	24.850	27.248
				.80	.7841	14.296	11.824	10.054	10.416
.05	.025	.0250	.6980	.70	.6937	47.193	31.731	21.747	22.984
				.80	.7919	12.634	10.426	8.850	8.901
.05	.05	.0000	−.0526	.70	.6753	48.146	32.349	22.118	26.592
				.80	.7690	12.904	10.672	9.070	10.103
.05	.05	.0250	.4737	.70	.6874	38.520	25.807	17.610	19.633
				.80	.7839	10.473	8.625	7.306	7.710
.05	.05	.05	1.0000	.70	.7000	32.341	21.663	14.750	15.273
				.80	.8000	8.936	7.327	6.185	6.141
.10	.05	.0000	−.0765	.70	.6626	34.051	22.709	15.386	20.736
				.80	.7539	9.386	7.725	6.535	7.911
.10	.05	.0250	.3059	.70	.6746	29.074	19.336	13.075	16.265
				.80	.7682	8.123	6.659	5.616	6.469
.10	.05	.0500	.6882	.70	.6870	25.565	16.953	11.439	13.174
				.80	.7835	7.233	5.905	4.964	5.417
.10	.10	.0000	−.1111	.70	.6497	26.612	17.622	11.833	18.020
				.80	.7388	7.530	6.170	5.198	6.800
.10	.10	.0500	.4444	.70	.6739	21.264	14.003	9.365	12.042
				.80	.7674	6.166	5.015	4.199	4.984
.10	.10	.1000	1.0000	.70	.7000	18.075	11.832	7.875	8.667
				.80	.8000	5.353	4.319	3.593	3.800
.15	.075	.0000	−.1196	.70	.6431	24.583	16.215	10.833	17.624
				.80	.7312	7.026	5.742	4.824	6.562
.15	.075	.0375	.2791	.70	.6611	20.930	13.749	9.159	13.019
				.80	.7521	6.093	4.952	4.142	5.248
.15	.075	.0750	.6778	.70	.6800	18.438	12.061	8.007	10.056
				.80	.7747	5.456	4.409	3.671	4.305
.15	.15	.0000	−.1765	.70	.6226	19.594	12.787	8.427	17.121
				.80	.7083	5.783	4.696	3.919	5.987
.15	.15	.0750	.4118	.70	.6595	15.617	10.110	6.621	9.920
				.80	.7502	4.760	3.826	3.167	4.179
.15	.15	.1500	1.0000	.70	.7000	13.396	8.594	5.583	6.507
				.80	.8000	4.189	3.329	2.728	3.038

Statement 2: In the last six months I have (not) sought help for a mental, emotional, or a psychological problem from a professional such as a psychiatrist, psychologist, or a social worker. Identify whether you belong to this group by saying "Yes" or "No."

The interviewing procedure was as follows. The subject entered the interview room. His (her) name was recorded, which, it was hoped, would make the subject take more seriously the sensitivity of the statements. Then the subject was randomly allocated to one of the two groups (direct response interview or M technique interview). In the case of the direct response interview the procedure was swift and simple and will not be elaborated on here. In the case of the M technique interview a sheet of paper bearing the two statements was handed to the respondent along with a deck of cards. The subject was asked to shuffle the deck well and draw a card at random (and not show it to the interviewer); the subject was asked to respond to the first statement (second statement) if the card came up spade, heart, or diamond (club), and his (her) response was recorded. The card was returned to the deck and the procedure was repeated, but the choices of statements was reversed this time; thus $P_{11}^{(1)} = P_{12}^{(2)} = .75$. The sheet of paper and the deck were then returned to the interviewer. Next, to assess the extent of preference of the M technique over the direct response technique, the following question was asked: "Supposing for the moment that your true response to either of the two statements was 'Yes,' would you feel more, less, or equally comfortable with this indirect method of questioning as compared to the direct method of questioning?" The response to this question was recorded and thus the interview was concluded. After the interview, the interviewers were asked to note down any unusual things (e.g., difficulty in understanding the instructions) that happened during the interview.

6.2 Results of the Application

Following is the summary of the responses obtained by using the two techniques:

Direct response: $n = 75$; No-No = 71, Yes-No = 3, No-Yes = 1, Yes-Yes = 0.

M technique: $n = 77$; No-No = 14, Yes-No = 5, No-Yes = 41, Yes-Yes = 17.

Thus from the direct response interviews we obtain the following estimates along with their standard errors (given inside the parentheses): $\hat{\theta}_1 = .04$ (.0226), $\hat{\theta}_2 = .0133$ (.0132), $\hat{\theta}_{12} = 0$ (.00).

To obtain the RMLE's of the θ's (by first obtaining the RMLE's of the π's) from the M technique interview data, we must maximize (3.5) subject to (3.4). For this purpose we used the generalized reduced gradient GRG algorithm of Abadie and Guigou (1969), which yielded the following estimates: $\hat{\theta}_1 = .05195$ (.0904), $\hat{\theta}_2 = .01300$ (.0774), $\hat{\theta}_{12} = .01039$ (.0562). The asymptotic standard errors of the estimates (given inside the parentheses) were computed from the formulas obtained by inverting the information matrix given in Section 3.4. The maximum value of the likelihood function was $L^* \propto (.314555)^{77}$. Note that in this case the RMLE's are the same as the UMLE's; that is, the UMLE's satisfy the constraints (3.4).

For testing $H_{\mathcal{F}}$ that the attributes 1 and 2 are uncorrelated, that is, $\theta_{12} = \theta_1\theta_2$, the GRG program was again run with the constraint set (3.6). This yielded the maximum value of the likelihood function under $H_{\mathcal{F}}$, namely $L_{\mathcal{F}}^* \propto (0.314479)^{77}$. The value of the χ^2 statistic works out to be .0372. Comparing this with the upper critical values of the chi-squared distribution with one df, we conclude that the null hypothesis of independence cannot be rejected. This small value of the χ^2 statistic is possibly due to two reasons: (a) we are dealing with rare attributes here and therefore much larger sample sizes are required to obtain a sufficiently powerful test; (b) in general, any RR technique yields a less powerful test compared with the direct response technique (assuming, of course, responses are equally truthful for both the techniques).

6.3 Discussion of the Results

First, we note that somewhat higher estimates of the θ's are obtained with the M technique than those obtained with the direct response technique, although the differences are not statistically significant. This might indicate that the respondents tend to be more truthful with the M technique interview. To the question (asked only of individuals in the M technique group) whether the respondent would feel more, equally, or less comfortable with the M technique than with the direct response technique, 43 responded that they would feel more comfortable, 29 responded that they would feel equally comfortable, and 5 responded they would feel less comfortable. These results show that the degree of truthfulness and cooperation by the respondents can be improved by using the M technique.

Finally, out of 77 respondents in the M technique group, about 5 respondents had some difficulty following the instructions and needed to go over the instructions one more time.

From this application we can conclude that the M technique is feasible in practice and is likely to improve the cooperation on the part of respondents and thus reduce the bias. Some care, however, is needed in explaining the instructions to the respondents.

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